Physics 445: Problem Set 3

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Due Friday, May 17, 2:30 p.m.

1. Start with the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \bar{\psi}\left(i\mathcal{D}^\mu\gamma_\mu - m\right)\psi - \frac{\xi}{2}B_aB^a + B_a\partial^\mu A^a_\mu + \bar{c}^a\left(-\partial^\mu\mathcal{D}^{ab}_\mu\right)c^b \tag{1}$$

and demonstrate that it is invariant under the BRST symmetry

$$\begin{aligned}
\delta A^a_\mu &= \epsilon\, \mathcal{D}^{ab}_\mu c^b \\
\delta\psi &= ig\,\epsilon\, c^a\, T_a\psi \\
\delta c^a &= -\frac{g}{2}\,\epsilon\, f^{abd}\, c^b c^d \\
\delta\bar{c}^a &= \epsilon\, B^a \\
\delta B^a &= 0
\end{aligned} \tag{2}$$

where ϵ is an infinitesimal anticommuting parameter.

2. Consider the supersymmetric extension of the Standard Model. For each chiral fermion (left or right) there is a complex scalar, which transform under the same representation of the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. For each gauge boson there is a Majorana fermion (half of degrees of freedon of Dirac fermion) transforming in the adjoint representation of the gauge group. In adittion, instead of one there are two Higgs fields, which transform in the fundamental representation of $SU(2)_L$, and which have hypercharges equal to 1/2 and $-1/2$ respectively. For each of the Higgs fields there is a chiral fermion transforming in the same representation of the gauge group as the Higgs fields.

a. Using the method of your choice, show that the contribution to the $SU(N)$ gauge coupling β-function of chiral fermions in a given representation of $SU(N)$ is equal to

$$\beta(g)_{\text{ch.f.}} = -g^3/(4\pi)^2(-2n_fC(r)/3)$$

.

This is just half of the contribution of Dirac fermions, and also applies to Majorana fermions.

b. Demonstrate that the contributions of scalars in the fundamental representation of $SU(N)$

is just half of the fermion one:

$$\beta(g)_{\text{scalars}} = \ -g^3/(4\pi)^2(-n_f/6)$$

d. Show that for $SU(N)$ with chiral fermions in the fundamental representation of the group, and n_H Higgs bosons in the fundamental representation, the result for the standard case is

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left(\frac{11}{3}N - \frac{n_f}{3} - \frac{n_H}{6}\right) \tag{3}$$

while, in the supersymmetric case, where chiral fermions and complex scalars come together, and the gauge bosons have an associated Majorana fermion in the adjoint representation of the group,

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left(3N - \frac{n_f}{2} - \frac{n_H}{2}\right) \tag{4}$$

c. Using these results, compute the $\beta(g)$ function of $SU(3)_c$ and $SU(2)_L$ in the Standard and in the Supersymmetric case.

d. Is asymptotic freedom preserved in all cases ? Considering the renormalization group evolution, at what scale do the $SU(3)_c$ and $SU(2)_L$ couplings acquire equal values ? ($\alpha_3(M_Z) = 0.118$, $\alpha_2(M_Z) = 0.0336$).

Extra credit Compute the $\beta(g)$ function of the $U(1)_Y$ group. Both the coupling and the associated β function in this case depend on an arbitrary normalization factor. What matters is the product of the coupling of the coupling and the charge, $g_1 Y$ of a given particle. The normalization, however, may be fixed if one assumes that the group $U(1)$ proceeds from the spontaneous breakdown of a bigger non-abelian group.
What should be the approximate normalization in order for the U(1) coupling α_1 to acquire the same value than α_2 and α_3 at the scale they meet ? Show that, for the supersymmetric case, this normalization is consistent with the one that would be obtained if the standard model group would proceed from the breakdown of a unification group $SU(5)$ at scales of the order of 10^{16} GeV.